2020 AMC10A

Problem 1

What value of x satisfies

$$x-\frac{3}{4}=\frac{5}{12}-\frac{1}{3}?$$

x 为何值时满足方程

$$x-\frac{3}{4}=\frac{5}{12}-\frac{1}{3}?$$

$\textbf{(A)}\ -\frac{2}{3} \qquad \textbf{(B)}\ \frac{7}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{2}{3} \qquad \textbf{(E)}\ \frac{5}{6}$

Problem 2

The numbers $3, 5, 7, a,$ and b have an average (arithmetic mean) of 15. What is the average of a and b?

数字 3，5，7，a 和 b 的平均值（算数平均值）是 15。则 a 和 b 的平均值是多少?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 45 \qquad \textbf{(E)}\ 60$

Problem 3

Assuming $a\neq 3$, $b\neq 4$, and $c\neq 5$, what is the value in simplest form of the following expression?

$$\frac{a-3}{5-c}\cdot\frac{b-4}{3-a}\cdot\frac{c-5}{4-b}$$

假设 $a\neq3$，$b\neq4$ 且 $c\neq5$，那么下式化成最简形式后的值为多少？

$$\frac{a-3}{5-c}\cdot\frac{b-4}{3-a}\cdot\frac{c-5}{4-b}$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ \frac{abc}{60} \qquad \textbf{(D)}\ \frac{1}{abc}-\frac{1}{60} \qquad \textbf{(E)}\ \frac{1}{60}-\frac{1}{abc}$

Problem 4

A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid $\$0.50$ per mile, and her only expense is gasoline at $\$2.00$ per gallon. What is her net rate of pay, in dollars per hour, after this expense?

一位司机以 60 英里每小时的速度开了 2 小时，在此期间她的车每消耗 1 加仑的汽油可以开 30 英里。她开 1 英里的收入是 0.5 美元，并且她的唯一支出是汽油支出，为每加仑 2 美元。在扣除掉此支出后，她的单位时间净收入是每小时多少美元？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 26$

Problem 5

What is the sum of all real numbers x for which $|x^2-12x+34|=2?$

满足方程$|x^2-12x+34|=2$的所有 x 的值之和为多少？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 25$

Problem 6

How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by $5?$

有多少个各个位上数字为偶数的 4 位正整数（即 1000 和 9999 之间的整数，包含 1000 和 9999）能被 5 整除？

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 200 \qquad \textbf{(E)}\ 500$

Problem 7

The 25 integers from -10 to $14,$ inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

从−10 到 14（包含−10 和 14）之间的 25 个整数可以放进一个 5 x 5 的正方形中，满足正方形的每一行，每一列以及每个主对角线的数字之和都相等。则这个共同的和为多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 50$

Problem 8

What is the value of

$$1+2+3-4+5+6+7-8+\cdots+197+198+199-200?$$

下式的值为多少

$$1+2+3-4+5+6+7-8+\cdots+197+198+199-200?$$

$\textbf{(A) } 9,800 \qquad \textbf{(B) } 9,900 \qquad \textbf{(C) } 10,000 \qquad \textbf{(D) } 10,100 \qquad \textbf{(E) } 10,200$

Problem 9

A single bench section at a school event can hold either 7 adults or 11 children. When N bench sections are connected end to end, an equal number of adults and children seated together will occupy all the bench space. What is the least possible positive integer value of $N?$

学校一次盛会中用的单个的一段长凳能够坐 7 个成人或者 11 个小孩，当 N 段这样的长凳首尾相连拼起来后，恰好相同数目的成人和小孩坐在一起能够坐满这些长凳。求最小的正整数 N 是多少？

$\textbf{(A) } 9 \qquad \textbf{(B) } 18 \qquad \textbf{(C) } 27 \qquad \textbf{(D) } 36 \qquad \textbf{(E) } 77$

Problem 10

Seven cubes, whose volumes are 1, 8, 27, 64, 125, 216, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

7 个体积分别为 1，8，27，64，125，216 和 343 的立方体竖直堆叠起来形成一座塔，且从塔底往上，立方体的体积依次减少。除了底部的立方体，其余立方体的底面恰好完全位于它下面的立方体的顶面上，这座塔的表面积（包括塔的底面）是多少？

$\textbf{(A) } 644 \qquad \textbf{(B) } 658 \qquad \textbf{(C) } 664 \qquad \textbf{(D) } 720 \qquad \textbf{(E) } 749$

Problem 11

What is the median of the following list of 4040 numbers?

$$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$$

下列 4040 个数的中位数是多少？

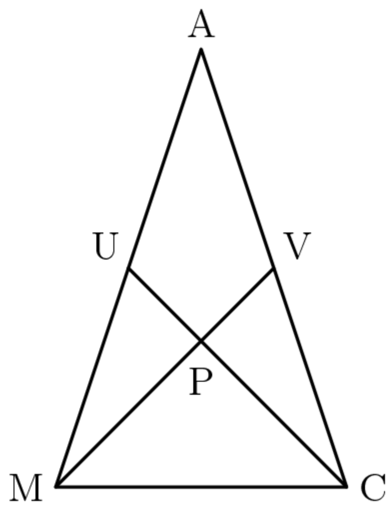
$$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$$

$\textbf{(A)}\ 1974.5 \qquad \textbf{(B)}\ 1975.5 \qquad \textbf{(C)}\ 1976.5 \qquad \textbf{(D)}\ 1977.5 \qquad \textbf{(E)}\ 1978.5$

Problem 12

Triangle AMC is isosceles with $AM = AC$. Medians $\overline{MV}$ and $\overline{CU}$ are perpendicular to each other, and $MV = CU = 12$. What is the area of $\triangle AMC$?

三角形 *AMC* 是个等腰三角形，*AM*=*AC*，中线$\overline{MV}$和$\overline{CU}$互相垂直，且 *MV*=*CU*=12，$\triangle AMC$ 的面积为多少？



$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 72 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 144 \qquad \textbf{(E)}\ 192$

Problem 13

A frog sitting at the point $(1,2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0,0), (0,4), (4,4),$ and $(4,0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

一只位于点（1，2）处的青蛙开始一系列跳跃，每一次跳跃都和坐标轴平行且跳跃长度为 1，跳跃的方向（向上，向下，向右或向左）随机选择．当青蛙达到以（0，0），（0，4），（4，4）和（4，0）为顶点的正方形的一条边上时，跳跃停止。问跳跃停止时，青蛙落在正方形的一条竖直的边上的概率为多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{5}{8} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{3}{4} \qquad \textbf{(E)}\ \frac{7}{8}$

Problem 14

Real numbers x and y satisfy $x + y = 4$ and $x \cdot y = -2$. What is the value of

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y?$$

实数 x 和 y 满足$x + y = 4$且$x \cdot y = -2$，则表达式的值是多少？

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y?$$

$\textbf{(A)}\ 360 \qquad \textbf{(B)}\ 400 \qquad \textbf{(C)}\ 420 \qquad \textbf{(D)}\ 440 \qquad \textbf{(E)}\ 480$

Problem 15

A positive integer divisor of $12!$ is chosen at random. The probability that the divisor chosen is a perfect square can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

从 12！的所有正整数因子中随机选择一个，则选出的这个因子是个完全平方数的概率可以写成 $\frac{m}{n}$，这里 m 和 n 是互质的正整数，$m+n$ 是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 23$

Problem 16

A point is chosen at random within the square in the coordinate plane whose vertices are $(0,0), (2020,0), (2020,2020),$ and $(0,2020)$. The probability that the point is within d units of a lattice point is $\frac{1}{2}$. (A point (x,y) is a lattice point if x and y are both integers.) What is d to the nearest tenth?

从坐标平面内以（0，0），（2020，0），（2020，2020）和（0，2020）为顶点的正方形内部随机选择一个点，这个点和格点距离不超过 d 的概率为$\frac{1}{2}$。（若 x 和 y 都是整数，那么点（x，y）称作格点）。那么 d 的值是多少（保留小数点后一位）？

(A) 0.3 **(B)** 0.4 **(C)** 0.5 **(D)** 0.6 **(E)** 0.7

Problem 17

Define

$$P(x) = (x-1^2)(x-2^2)\cdots(x-100^2).$$

How many integers n are there such that $P(n) \leq 0$?

定义

$$P(x) = (x-1^2)(x-2^2)\cdots(x-100^2).$$

有多少个整数 n 满足$P(n) \leq 0$?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100

Problem 18

Let (a,b,c,d) be an ordered quadruple of not necessarily distinct integers, each one of them in the set $0,1,2,3.$ For how many such quadruples is it true that $a\cdot d-b\cdot c$ is odd? (For example, $(0,3,1,1)$ is one such quadruple, because $0\cdot 1-3\cdot 1=-3$ is odd.)

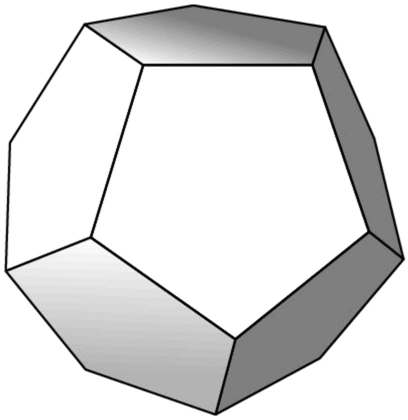
（a，b，c，d）是个有序的四元组，它的每个数字都是集合｛0，1，2，3｝中的元素且不必都不相同，有多少个这样的四元组满足$a\cdot d-b\cdot c$是奇数？（如（0，3，1，1）就是这样一个四元组因为$0\cdot 1-3\cdot 1=-3$是奇数）

(A) 48 **(B)** 64 **(C)** 96 **(D)** 128 **(E)** 192

Problem 19

As shown in the figure below, a regular dodecahedron (the polyhedron consisting of 12 congruent regular pentagonal faces) floats in empty space with two horizontal faces. Note that there is a ring of five slanted faces adjacent to the top face, and a ring of five slanted faces adjacent to the bottom face. How many ways are there to move from the top face to the bottom face via a sequence of adjacent faces so that each face is visited at most once and moves are not permitted from the bottom ring to the top ring?

如下图所示，一个正十二边形（一共由 12 个全等的正五边形组成的立体图形）悬浮在空间中且上下底面水平。注意到和顶面相邻的是 5 个斜面形成的环（称为顶环），和底面相邻的面也是 5 个斜面形成的环（称为底环），想沿着一系列相邻的面从顶面移动到底面，满足每个面最多访问一次且不允许从底环移动到顶环的方法数一共有多少种？



(A) 125 **(B)** 250 **(C)** 405 **(D)** 640 **(E)** 810

Problem 20

Quadrilateral $ABCD$ satisfies $\angle ABC = \angle ACD = 90^\circ$, $AC = 20$, and $CD = 30$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E, and $AE = 5$. What is the area of quadrilateral $ABCD$?

四边形 *ABCD* 满足$\angle ABC = \angle ACD = 90^\circ$, $AC = 20$, $CD = 30$. 对角线$\overline{AC}$和$\overline{BD}$交于点 *E*，且$AE = 5$.。四边形 *ABCD* 的面积为多少?

(A) 330 **(B)** 340 **(C)** 350 **(D)** 360 **(E)** 370

Problem 21

There exists a unique strictly increasing sequence of nonnegative integers $a_1 < a_2 < \ldots < a_k$ such that

$$\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}.$$

What is k?

存在唯一一个严格递增的非负整数组成的数列$a_1 < a_2 < \ldots < a_k$满足

$$\frac{2^{289}+1}{2^{17}+1} = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}.$$

则 k 是多少？

(A) 117 **(B)** 136 **(C)** 137 **(D)** 273 **(E)** 306

Problem 22

For how many positive integers $n \le 1000$ is $\left\lfloor \frac{998}{n} \right\rfloor + \left\lfloor \frac{999}{n} \right\rfloor + \left\lfloor \frac{1000}{n} \right\rfloor$ not divisible by 3?

(Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to x.)

有多少个正整数 n，$n \leqslant 1000$，满足$\left\lfloor \frac{998}{n} \right\rfloor + \left\lfloor \frac{999}{n} \right\rfloor + \left\lfloor \frac{1000}{n} \right\rfloor$不能被 3 整除？（记住，符号$\lfloor x \rfloor$表示取小于或等于 x 的最大整数）

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

Problem 23

Let T be the triangle in the coordinate plane with vertices $(0,0)$, $(4,0)$, and $(0,3)$. Consider the following five isometries (rigid transformations) of the plane: rotations of 90°, 180°, and 270° counterclockwise around the origin, reflection across the x-axis, and reflection across the y-axis. How many of the 125 sequences of three of these transformations (not necessarily distinct) will return T to its original position? (For example, a 180° rotation, followed by a reflection across the x-axis, followed by a reflection across the y-axis will return T to its original position, but a 90° rotation, followed by a reflection across the x-axis, followed by another reflection across the x-axis will not return T to its original position.)

T是坐标平面内以（0，0），（4，0）和（0，3）为顶点的三角形考虑如下 5 种平面变换：绕着原点逆时针旋转 90°，180°，270°，关于 x 轴作对称，关于 y 轴作对称，从这 5 种变换中选择 3 种变换（不必要不同），一共可以组成 125 种这样的 3 次变换。这 125 种变换中，有多少种可以将 T 再变换回它原来的位置？（例如，180° 旋转后关于 x 轴对称，接着是关于 y 轴对称将把 T 再变换回它原来的位置，但是 90° 旋转后关于 x 轴对称，接着再关于 x 轴作对称将不能把 T 变换回它原来的位置。）

(A) 12 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 25

Problem 24

Let n be the least positive integer greater than 1000 for which

$$\gcd(63, n+120) = 21 \quad \text{and} \quad \gcd(n+63, 120) = 60.$$

What is the sum of the digits of n?

n 是大于 1000 的最小正整数，满足

$$\gcd(63, n+120) = 21 \quad \text{and} \quad \gcd(n+63, 120) = 60.$$

n 的各个位上的数字之和为多少？

(A) 12 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 24

Problem 25

Jason rolls three fair standard six-sided dice. Then he looks at the rolls and chooses a subset of the dice (possibly empty, possibly all three dice) to reroll. After rerolling, he wins if and only if the sum of the numbers face up on the three dice is exactly 7. Jason always plays to optimize his chances of winning. What is the probability that he chooses to reroll exactly two of the dice?

Jason 扔了 3 个标准的六面骰子。当他看到投出的结果后，再决定选择其中一部分骰子（可能都不选，可能 3 个骰子都选）重新再扔一次。他第二次扔完后，当且仅当 3 个骰子面朝上的数字之和恰好为 7 时他才赢。Jason 总是设法使得自己赢的概率最大，他恰好选择其中 2 个骰子重新扔的概率是多少?

$\textbf{(A)}\ \frac{7}{36} \qquad \textbf{(B)}\ \frac{5}{24} \qquad \textbf{(C)}\ \frac{2}{9} \qquad \textbf{(D)}\ \frac{17}{72} \qquad \textbf{(E)}\ \frac{1}{4}$

2020 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
E	C	A	E	C	B	C	B	B	B	C	C	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	E	B	E	C	E	D	C	A	A	C	A	

2020 AMC 10A Solution

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